Corrections and Explanations for Luan Hoang's publications

April 28, 2026

A. The Navier–Stokes Equations

1. Paper [7], formula (1.13): remove the last "$= 0$" (for B_j).

2. Paper [16], sentence after (4.58): insert "u_{N+1}" after "since", and add the terms "multiplied by $e^{-\mu_{N+1}t}$" after "G_{μ,σ_0}-valued polynomial" .

   Explanations: Summing up (4.58) is permitted by simply working with the coefficients of the polynomials.

3. Paper [17]:

   (a) Page 113, line 3: replace "linear" with "bilinear"

   (b) After (3.15), in the formula of ξ_1: remove $e^{\mu_1\tau}$ in the integral.

   (c) Before (3.33), in the formula of ξ_{N+1}: remove $e^{\mu_{N+1}\tau}$ in the integral.

4. Paper [8]:

   (a) The end of (2.11): $\mathbb{R}^n$ should be $\mathbb{R}^3$.

   (b) At the bottom of page 990, repalce $(e^{-(\mu_N+\varepsilon_N/2)t}))$ with $\mathcal{O}(e^{-(\mu_N+\varepsilon_N/2)t})$

5. Paper [9]:

   (a) Page 5, after (2.4): replace "eigenvector" with "eigenvalue".

   (b) Page 20, line 4: change $\mathcal{E}_{\mathbb{K}}(m,k,-\mu)$ to $\mathcal{E}_{\mathbb{C}}(m,k,-\mu)$

   (c) Page 21, line 5: change $\mathcal{E}_{\mathbb{C}}(m,N,-\mu)$ to $\mathcal{E}_{\mathbb{C}}(m,k,-\mu)$

   (d) Page 21, line 6: change $\mathcal{E}_{\mathbb{K}}(m,N,-\mu)$ to $\mathcal{E}_{\mathbb{C}}(m,k,-\mu)$

   (e) Page 25, after (5.25): $(\mathcal{Z}_{A_{\mathbb{C}}}p)... \in G_{\alpha+1,\sigma}$ should be $(\mathcal{Z}_{A_{\mathbb{C}}}p)... \in G_{\alpha+1,\sigma,\mathbb{C}}$

   (f) Page 30, 2nd line after (6.30): replace $T_* > E_k(0)$ with $u(t)$ is a Leray-Hopf weak solution on $[T_*,\infty)$

   (g) Page 31, (6.33): replace C with C^2

   (h) Page 35, lines -1 and -2: $\sum_{k=1}^N$ should be $\sum_{\lambda=1}^N$

6. Paper [12]:

(a) Page 18, the third line above Theorem 4.3: the citation [3, lemma 3.1] should be [3, lemma 3.2]

B. Porous Media Equations

1. Paper [1], formula (37) and line 1 of the next page: replace $(K(\xi)\xi^n)$ with derivative $(K(\xi)\xi^n)'$

2. Paper [10], Lemma 2.3, inequality (2.24) and first inequality of part (ii): replace "$\geq a$" with "$\geq (1-a)$"

3. Paper [15]:

   (a) Page 279, line 3: insert "$+2s$" in front of the integral.

   (b) Page 330, the line above (A.10): $\widetilde{A}$ should be $(A_1 D^{\mu_1} + A_2 D^{\mu_2})/D^{\mu_1}$.

4. Paper [14]:

   (a) Page 17, inequalities (128) and (132): "$-\inf|w(x,0)|$" should be "$\inf(-|w(x,0)|)$" and, hence, "$-\sup|w(x,0)|$".

5. Paper [5]:

   (a) Lemma A.1: Because $\prod \gamma_j$ is convergent and by the Cauchy criterion, the sequence $(G_j)_{j=1}^{\infty}$ is bounded. Hence number G is finite.

6. Paper [4]:

   (a) Inequalities (3.11) on page 3616 and (3.44) on page 3631, insert constant $\alpha(\alpha-\lambda)d_3/(8\lambda)$ before the integral $\int_U |\nabla u|^{2-a}u^{\alpha-\lambda-1}dx$ on the left.

   (b) Page 3629, after (3.28), reference (3.25) should be (3.28).

   (c) Line 3 from the bottom of page 3632, line 3 of page 3633, and line 2 of page 3635: The constant C should be $\bar{C}$.

   (d) Page 3633, inequality (4.6): the last two terms should be multiplied by 2.

   (e) In inequalities (4.8) on page 3633 and (4.9) on page 3634, the integrand of the integrals $\iint_{Q_T} u^{\alpha-\lambda-1}|\nabla u|^{2-a}dxdt$ should be multiplied by ξ^2.

   (f) Page 3634, after (4.11): value of ε should be $(\alpha-\lambda)d_3/32$.

7. Paper [6]:

(a) Page 955, in the formulas of c_4 after (2.12), and of c_6, c_7, c_8 after (2.15), replace α_N with $\bar{\alpha}_N$.

8. Paper [13]:

   (a) Page 2, line 2: "may" should be "many"

C. Ordinary Differential Equations

1. Paper [2]:

   (a) Page 1195: In (3.5), $\varepsilon_0 = C_0/2$.

2. Paper [3]:

   (a) Page 17: 2nd line after (4.25), $\sum_{j=1}^m \mu_{k_j}$ should be $\sum_{j=1}^m \widetilde{\mu}_{k_j}$

   (b) Page 17: after (4.26), "condition (4.27)" should be "condition (4.26)"

   (c) Page 20, last line, and page 21, lines 3 and 6: "z_N" should be "z_m".

D. Partial differential equations in non-divergence form

1. Paper [11]:

   (a) Page 7, line 5: add "for" to the beginning of the line.

References

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[2] D. Cao and L. Hoang. Asymptotic expansions with exponential, power, and logarithmic functions for non-autonomous nonlinear differential equations. *J. Evol. Equ.*, 21(2):1179–1225, 2021.

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[4] E. Celik, L. Hoang, and T. Kieu. Doubly nonlinear parabolic equations for a general class of Forchheimer gas flows in porous media. *Nonlinearity*, 31(8):3617–3650, 2018.

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[7] C. Foias, L. Hoang, and J.-C. Saut. Asymptotic integration of Navier-Stokes equations with potential forces. II. An explicit Poincaré-Dulac normal form. *J. Funct. Anal.*, 260(10):3007–3035, 2011.

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[12] L. Hoang and M. S. Jolly. Intrinsic expansions in large Grashof numbers for the steady states of the Navier–Stokes equations. *Nonlinearity*, 37(12):Paper No. 125008, 32 pp, 2024.

[13] L. Hoang and T. Kieu. Anisotropic flows of Forchheimer-type in porous media and their steady states. *Nonlinear Anal. Real World Appl.*, 84:Paper No. 104269, 30 pp, 2025.

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[15] L. T. Hoang, T. T. Kieu, and T. V. Phan. Properties of generalized Forchheimer flows in porous media. *J. Math. Sci.*, 202(2):259–332, 2014.

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[17] L. T. Hoang and E. S. Titi. Asymptotic expansions in time for rotating incompressible viscous fluids. *Ann. Inst. H. Poincaré Anal. Non Linéaire*, 38(1):109–137, 2021.